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THIN CYLINDRICAL SHELLS UNDER LOCAL AXIAL LOADINGS

TECHNICAL REPORT

by

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DECEMBER 1964

MATERIALS ENGINEERING DIVISION
U. S. ARMY MATERIALS RESEARCH AGENCY
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Elastic shells
Deformation
Stresses

THIN CYLINDRICAL SHELLS UNDER LOCAL AXIAL LOADINGS

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ABSTRACT

The linear equations due to Flügge for thin elastic circular cylindrical shells are solved by use of the finite Fourier transform for the case of distributed tangential (axial) loading with simply supported edge conditions. This solution supplements prior results obtained by Bijlaard for radial (normal) distributed loadings.

Combining Bijlaard's results with the solutions obtained in this report permits discussion of localized loadings having components both normal and tangential to the shell's middle surface. Such an application is discussed and numerical results are provided.

CONTENTS

INTRODUCTION

It frequently happens in pressure vessel and missile construction that an axial load is transmitted into the wall of a thin cylindrical shell by means of a heavy ring attached to the cylinder. This ring distributes the load fairly uniformly into the cylinder. For reasons of economy or weight-saving, it is sometimes desirable to replace the solid ring by a series of brackets or angle irons. The resulting nonuniform distribution of axial load can result in a significant increase in the local stress level in the cylinder. This situation thus provides an interesting and useful problem in the asymmetric deformation of thin shells. Solutions and numerical results for similar localized radial (normal) and tangential (in the circumferential direction) loadings on simply supported cylinders have been presented by Bijlaard [1,2]. However, the related problem of a distributed tangential loading in the axial direction was not discussed. It is our intention, then, to supplement these results by examining the solution and presenting some numerical results for localized axial loadings on thin cylindrical shells.

In the specific application described earlier, the axial load is not applied directly to the shell's middle surface but usually at a distance greater than half the thickness away from the middle surface. Thus, there is also developed a radial load with an axial variation, i.e., a longitudinal moment, to use Bijlaard's nomenclature. The desired solution to the physical problem is then a properly weighted combination of these two loading conditions.

In the ensuing discussion, the governing differential equations of equilibrium selected are those due to Flügge rather than those employed by Bijlaard. This approach affords, as a by-product, an opportunity to compare results of Flügge's nonhomogeneous equations with those of Bijlaard's for a wide range of shell parameters. Hoff [3] and Kempner [4] have made similar comparisons with Donnell's equations for the corresponding homogeneous equations and for line loadings.

Recently, Nash and Bridgland [5] discussed line radial loadings on thin cylindrical shells using Flügge's equations and suggested the use of the finite Fourier transform in obtaining solutions. This technique has been employed in the present discussion of distributed axial loads.

BASIC EQUATIONS AND FORMULATION OF PROBLEM

We consider a thin circular cylindrical shell of radius a and thickness h. Young's modulus and Poisson's ratio are denoted by E and ν, respectively, while x^*, θ, and Z denote coordinates in the axial, circumferential, and radial (positive when inward) directions. Corresponding displacement components for a point on the shell's middle surface are u^*, v^*, and w^*. It is convenient to deal with the dimensionless displacement components u, v, and w, defined by $u = u^*/a$, $v=v^*/a$, and $w=w^*/a$, as well as with dimensionless middle-surface coordinates $x=x^*/a$ and θ.

Flügge's equations for the equilibrium of thin circular cylindrical shells [6] may then be written in the dimensionless form

$$u'' + \left(\frac{1-\nu}{2}\right)\ddot{u} + \left(\frac{1+\nu}{2}\right)\dot{v}' - \nu w' + k^2\left[w''' + \left(\frac{1-\nu}{2}\right)(\ddot{u} - \ddot{w}')\right] = -\frac{ap_x}{C}$$

$$\ddot{v} + \left(\frac{1-\nu}{2}\right)v'' + \left(\frac{1+\nu}{2}\right)\dot{u}' - \dot{w} + k^2\left[\left(\frac{3-\nu}{2}\right)\dot{w}'' + \frac{3(1-\nu)}{2}v''\right] = -\frac{a^2p_\theta}{C} \qquad (1)$$

$$\nabla^4 w + 2\ddot{w} + w + \left(\frac{3-\nu}{2}\right)\dot{v}'' + u''' - \left(\frac{1-\nu}{2}\right)\ddot{u}' + \frac{1}{k^2}\left[w - \dot{v} - \nu u'\right] = \frac{a^2p_r}{D}$$

where dots indicate differentiation with respect to θ, and primes indicate differentiation with respect to x; p_x, p_θ, and p_r are the components of applied surface loading in the axial, circumferential, and radial directions, $k^2 = h^2/12a^2$, $C = Eh/(1-\nu^2)$, $D = Eh^3/12(1-\nu^2)$, and $\nabla^4 w \equiv w'''' + 2\ddot{w}'' + \ddddot{w}$.

When Equations 1 have been solved for u, v, and w for a prescribed loading condition, stress resultants and moments in the cylinder may be obtained from the following relations:

$$
\begin{aligned}
N_x &= C\,[u' + \nu\,(\dot{v} - w)] + \bar{D}\,w'' \\
N_\theta &= C\,[\dot{v} + \nu u' - w] - \bar{D}\,[w + \ddot{w}] \\
N_{x\theta} &= C\,(\tfrac{1-\nu}{2})\,[\dot{u} + v'] + \bar{D}\,(\tfrac{1-\nu}{2})[v' + \dot{w}'] \\
N_{\theta x} &= C\,(\tfrac{1-\nu}{2})\,[\dot{u} + v'] + D\,(\tfrac{1-\nu}{2})[\dot{u} - \dot{w}'] \\
\\
M_x &= -\bar{D}a\,[w'' + \nu\,\ddot{w} + u' + \nu\,\dot{v}] \\
M_\theta &= -\bar{D}a\,[w + \ddot{w} + \nu\,w''] \\
M_{x\theta} &= -\bar{D}a\,(1-\nu)[\dot{w}' + v'] \\
M_{\theta x} &= -\bar{D}a\,(1-\nu)[\dot{w}' + \tfrac{1}{2}\,(v' - \dot{u})]
\end{aligned}
\qquad (2)
$$

$$\bar{D} \equiv D/a^2.$$

We consider the following specific loading and boundary conditions. An axial load of intensity p_x is distributed over a set of N rectangular areas (pads) periodically located around the circumference of the cylinder (see Figure 1). Let the length in the axial direction of each pad be $2a\delta$ and the x coordinate of the center of each pad be x_o; let the length of each pad in the θ direction be $2a\theta_1$, while the θ coordinate of the center of the first pad is $\theta=0$. The distributed applied load is assumed to be balanced by a uniform axial compression N_x at the base of the cylinder. This desired loading system may be considered as the superposition of the following two systems:

1. a rotationally symmetric distribution of uniform axial load acting over a band of length $2a\delta$, centered about $x=x_o$, and balanced by a uniform compression N_x at the base of the cylinder.

2. a self-equilibrating distribution of tangential axial load acting over the same area (see Figure 2).

The solution to the first of these systems is trivial. Hence, the given problem is reduced to the solution of equilibrium equations when $p_\theta = p_r = 0$ and $p_x(x,\theta)$ has the form of a step function in x for $\theta=\theta_o$ and a rectangular wave function in θ for $x_o - \delta \le x \le x_o + \delta$ (see Figure 3).

Algebraically, then,

$$p_x(x,\theta) = p_1 \qquad 2(m-1)(\theta_1+\theta_2) - \theta_1 \le \theta \le 2(m-1)(\theta_1+\theta_2) + \theta_1$$

$$= -p_2 \qquad 2(m-1)(\theta_1+\theta_2) + \theta_1 \le \theta \le 2m(\theta_1+\theta_2) - \theta_1 \tag{3}$$

where $m=1,2,\ldots N$, $\theta_1+\theta_2 = \pi/N$, $x_o - \delta \le x \le x_o + \delta$.

For x outside this range, p_x vanishes identically. The condition that the load be self-equilibrating is that $p_1\theta_1 = p_2\theta_2$. Finally, we consider boundary conditions of simple support at the ends, i.e., $w = v = N_x = M_x = 0$ at $x=0$, L/a.

METHOD OF SOLUTION

We assume the following forms for the displacements:

$$w = \sum_{n=1}^{\infty} F_{1n}(\theta) \sin(\lambda x)$$

$$v = \sum_{n=1}^{\infty} F_{2n}(\theta) \sin(\lambda x) \tag{4}$$

$$u = F_{3o}(\theta) + \sum_{n=1}^{\infty} F_{3n}(\theta) \cos(\lambda x)$$

where $\lambda = \dfrac{n\pi a}{L}$.

This form ensures simply supported edges at $x=0$, L/a. We further expand the applied load $p_x(x,\theta)$ in a Fourier cosine series in x, obtaining:

$$p_x(x,\theta) = \frac{a_o(\theta)}{2} + \sum_{n=1}^{\infty} a_n(\theta) \cos(\lambda x)$$

$$a_n(\theta) = \frac{4}{n\pi} p(\theta) \cos(\lambda x_o) \sin(\lambda\delta) \tag{5}$$

$$a_o(\theta) = \frac{4\delta a p(\theta)}{L}.$$

Insertion of these expansions into the equilibrium equations (1) leads to a set of 3n ordinary differential equations for $F_{in}(\theta)$ whose solution may rather compactly be obtained by means of the finite Fourier transform. The essential facts regarding this transform may be listed as follows. The finite Fourier transform of a function $f(\theta)$ is defined as:

$$f^*(S) = F[f(\theta)] = \frac{1}{2\pi} \int_0^{2\pi} f(\theta)\, e^{-iS\theta}\, d\theta .$$

The inverse transform is given by

$$f(\theta) = F^{-1}[f^*(S)] = \sum_{S=-\infty}^{\infty} f^*(S)\, e^{iS\theta} ,$$

while the following useful relation holds [7]

$$F\left[\frac{df}{d\theta}\right] = iS\, f^*(S).$$

Application of the transform then reduces the equilibrium equations to a set of 3n algebraic equations to be solved for the transforms of $F_{mn}(\theta)$:

$$F_{1n}^*(S)\left\{k^2q^4 + 2k^2q^2(1-\lambda^2) + [1+k^2(1+\lambda^4)]\right\} + F_{2n}^*(S)\left\{-q[1+k^2\lambda^2(\tfrac{3-\nu}{2})]\right\}$$
$$+ F_{3n}^*(S)\left\{\lambda k^2q^2(\tfrac{1-\nu}{2}) + \lambda(\nu+\lambda^2k^2)\right\} = 0$$

$$F_{1n}^*(S)\left\{-q\,[1+\lambda^2k^2(\tfrac{3-\nu}{2})]\right\} + F_{2n}^*(S)\left\{q^2-\lambda^2(\tfrac{1-\nu}{2})(1+3k^2)\right\}$$
$$+ F_{3n}^*(S)\left\{-q\lambda(\tfrac{1+\nu}{2})\right\} = 0 \qquad (6)$$

$$F_{1n}^*(S)\left\{q^2\lambda k^2(\tfrac{1-\nu}{2}) + \lambda(\nu+\lambda^2k^2)\right\} + F_{2n}^*(S)\left\{-\lambda q(\tfrac{1+\nu}{2})\right\}$$
$$+ F_{3n}^*(S)\left\{\lambda^2-q^2(1+k^2)(\tfrac{1-\nu}{2})\right\} = \frac{a(a_n)}{C} F\left\{p(\theta)\right\}$$

where $q \equiv iS$.

The transform of $p(\theta)$, the θ-dependence of the load function is easily found to be

$$F\,[p(\theta)] = \frac{1}{\pi S}\left\{p_1 \sin (S\,\theta_1) \sum_{m=1}^{N} e^{-2(m-1)(\theta_1+\theta_2)\,iS}\right.$$

$$\left. -p_2 \sin (S\,\theta_2) \sum_{m=1}^{N} e^{-(2m-1)(\theta_1+\theta_2)\,iS}\right\} \qquad (7)$$

$$= \frac{N}{\pi S}\left\{p_1 \sin (S\,\theta_1) - (-1)^k\, p_2 \sin (S\,\theta_2)\right\} \qquad s = kN \quad k=1,2,3\ldots$$

$$= 0 \qquad s \neq kN.$$

With this information, simple algebra permits an explicit solution for the transforms of $F_{mn}(\theta)$. Inserting these into the inversion formula and taking account of the odd or even character of the resulting expressions leads directly to the following results for $F_{mn}(\theta)$:

$$F_{mn}(\theta) = \frac{2a\,(a_n)\,N(p_1+p_2)}{C\pi k^2} \sum_{S=N,2N\ldots}^{\infty} \frac{A_{mn}(S)\,\sin(S\,\theta_1)}{S\,B_n(S)} \cos (S\theta) \qquad m=1,3 \qquad (8)$$

$$F_{2n}(\theta) = \frac{2a\,(a_n)\,Ni(p_1+p_2)}{C\pi k^2} \sum_{S=N,2N\ldots}^{\infty} \frac{A_{2n}(S)\,\sin (S\,\theta_1)}{S\,B_n(S)} \sin (S\theta)$$

where

$$A_{1n}(S) = \left\{S^4\lambda k^2 + S^2[3\lambda^3 k^4(\tfrac{1-\nu}{2}) + \lambda] - (1+3k^2)(\lambda^5 k^2 + \nu\lambda^3)\right\}$$

$$A_{2n}(S) = iS\left\{-S^4 k^2\lambda\left(\tfrac{1+\nu}{1-\nu}\right) - S^2\left[\lambda^3\left(2k^2\left(\tfrac{1+\nu}{1-\nu}\right) + \tfrac{3-\nu}{2}k^4\right) - \lambda k^2\left(\tfrac{1-3\nu}{1-\nu}\right)\right]\right.$$

$$+ \frac{2}{1-\nu}\left[\lambda^5\left(k^4\left(\tfrac{3-\nu}{2}\right) - k^2\left(\tfrac{1+\nu}{2}\right)\right) + \lambda^3 k^2\left(1+\left(\tfrac{3-\nu}{2}\right)\nu\right)\right.$$

$$\left.\left. + \lambda\left(\nu - \frac{1+\nu}{2}(1+k^2)\right)\right]\right\}$$

$$A_{3n}(S) = \frac{2k^2}{1-\nu}\left\{S^6 + S^4\left[-2 + \frac{\lambda^2}{2}\left(5-\nu + 3k^2(1-\nu)\right)\right]\right.$$

$$+ S^2\left[\lambda^4\left((2-\nu) + k^2\left(3(1-\nu) - \left(\tfrac{3-\nu}{2}\right)^2\right)\right)\right.$$

$$\left. - \lambda^2\left(4-2\nu + k^2(3-3\nu)\right) + 1\right]$$

$$\left. + \left[\lambda^6(1+3k^2)\left(\tfrac{1-\nu}{2}\right) + \lambda^2(1+k^2)(3+1/k^2)\left(\tfrac{1-\nu}{2}\right)\right]\right\} \tag{9}$$

$$B_n(S) = \left\{S^8(1+k^2) + S^6\left[-2(1+k^2) + \lambda^2\left(4+k^2\left(\tfrac{7-3\nu}{2}\right) + k^4\left(\tfrac{3(1-\nu)}{2}\right)\right)\right]\right.$$

$$+ S^4\left[\lambda^4\left(6 + 3k^2(2-\nu) - \nu^2 k^4\right)\right.$$

$$\left. - \lambda^2\left(2(4-\nu) + k^2(7-5\nu) + k^4\, 3(1-\nu)\right) + (1+k^2)\right]$$

$$+ S^2\left[\lambda^6\left(4 + \frac{(11-3\nu)}{2}k^2 + \frac{9(1-\nu)}{2}k^4\right)\right.$$

$$- 3\lambda^4\left(2 + (2-\nu+\nu^2)k^2\right)$$

$$\left. + \lambda^2\left(2(2-\nu) + \frac{7(1-\nu)}{2}k^2 + \frac{3(1-\nu)k^4}{2}\right)\right]$$

$$\left. + (1+3k^2)\lambda^4\left[1 + \frac{1-\nu^2}{k^2} - 2\nu\lambda^2 + (1-k^2)\lambda^4\right]\right\}.$$

Equations (8) and (4) thus provide representations for the displacements u, v, w, and so a solution to the problem in the form of a double Fourier series in x and θ. It is readily established that these series for displacements converge and, in fact, sufficiently rapidly that they may be differentiated to provide convergent series for the stress resultants and couples. To complete the stress analysis of the cylinder under tangential axial loadings over a series of N pads, it is necessary to superimpose upon the stresses due to the above self-equilibrating load system those due to a rotationally symmetric band of tangential axial loads (see Figure 2). We must, therefore, add to N_x the terms:

$$\begin{aligned}
\bar{N}_x &= 0 & 0 \le x \le x_o - \delta \\
&= \frac{-N_o(x-x_o+\delta)}{2\delta} & x_o-\delta \le x \le x_o + \delta \quad (\text{all } \theta) \qquad (10)\\
&= -N_o & x_o+\delta \le x \le L
\end{aligned}$$

where $N_o = 2\delta a p_2$. Finally, we note that the intensity of $p_x(x,\theta)$ over the periodic pad areas then has a magnitude of p_1+p_2.

LONGITUDINAL MOMENT SOLUTION

To account for the fact that the axial load is not applied at the middle surface of the shell, we assume that there is a net longitudinal moment exerted over the area beneath each pad, and that this moment may be described in terms of a radial pressure distribution whose intensity is proportional to its algebraic distance from x_o:

$$p_r(x,\theta) = \bar{p}(\theta)\,(x-x_o) \qquad x_o-\delta \le x \le x_o+\delta$$

where $\bar{p}(\theta)$ has a unit value, say p_o, beneath a pad and vanishes elsewhere.

The solution for this type of loading may be obtained most expediently by utilizing Nash and Bridgland's[5] basic results for a radial line load acting over a portion of the circumference at $x=\xi$. Their solution for this problem may be written in the form

$$w = \sum_{n=1}^{\infty} F_{1n}(\theta) \sin(\lambda x)$$

$$v = \sum_{n=1}^{\infty} F_{2n}(\theta) \sin(\lambda x) \tag{11}$$

$$u = \sum_{n=1}^{\infty} F_{3n}(\theta) \cos(\lambda x),$$

where

$$F_{mn}(\theta) = \frac{2(1-\nu^2)a^2}{Eh\ k^2L} \sin(\lambda\xi) \left\{p^*(0)E_{mn}{}^*(0) + 2\sum_{\substack{S=1 \\ m=1,3}}^{\infty} p^*(S)E_{mn}{}^*(S)\cos S\theta\right\}$$

$$F_{2n}(\theta) = \frac{2(1-\nu^2)a^2}{Eh\ k^2L} \sin(\lambda\xi) \left\{2i\sum_{S=1}^{\infty} p^*(S)\ E_{2n}{}^*(S)\ \sin S\theta\right\}, \tag{12}$$

$p^*(S)$ is the finite Fourier transform of $p(\theta)$,

$$E_{mn}{}^*(S) = \frac{\bar{A}_{mn}(S)}{\bar{B}_m(S)} \tag{13}$$

and

$$\bar{A}_{1n}(S) = S^4(1+k^2) + \lambda^2 S^2\ [2+2\ (1-\nu)k^2 + 3(\frac{1-\nu}{2})\ k^4] + \lambda^4[1+3k^2]$$

$$\bar{A}_{2n}(S) = -iS\left\{S^2\ [(1+k^2) + k^2\lambda^2\ (2+\frac{3-\nu}{2}\ k^2)] + \lambda^2\ [2+\nu+2k^2\lambda^2]\right\} \tag{14A}$$

$$\bar{A}_{3n}(S) = \lambda\left\{k^2S^4 + S^2\ [1+3\ (\frac{1-\nu}{2})\ k^2\ \lambda^2] - (1+3k^2)[\nu+k^2\lambda^2]\ \lambda^2\right\}$$

$$\bar{B}_n(S) = S^8\,(1+k^2) + S^6 \left\{ \lambda^2\, [4 + \frac{7-3\nu}{2}\, k^2 + \frac{3(1-\nu)}{2}\, k^4] - 2(1+k^2) \right\}$$

$$+ S^4 \left\{ \lambda^4\, [6+3(2-\nu)k^2 - \nu^2 k^4] - \lambda^2 [2(4-\nu)+(7-5\nu)k^2+3(1-\nu)k^4] + (1+k^2) \right\}$$

$$+ \lambda^2 S^2 \left\{ \lambda^4 [4+(\frac{11-3\nu}{2})\, k^2 + 9\,(\frac{1-\nu}{2})\, k^4] - 3\lambda^2\, [2+(2-\nu+\nu^2)\, k^2] \right. \qquad (14B)$$

$$\left. + [2\,(2-\nu) + 7\,(\frac{1-\nu}{2})\, k^2 + 3\,(\frac{1-\nu}{2})\, k^4] \right\}$$

$$+ (1+3k^2)\, \lambda^4\, [1 + \frac{1-\nu^2}{k^2} - 2\nu\lambda^2 + (1-k^2)\, \lambda^4].$$

We may treat this solution as a kernel and integrate suitably to obtain the desired solution for the radial load with axial variation. For example, if the displacement w of Reference 5 is denoted momentarily by $w_5(x,\theta,\xi)$, then the displacement $w(x,\theta)$ due to a band of radial pressure varying linearly about x_o and within a band of length $2a\delta$ is given by

$$w\,(x,\theta) = \int_{x_o-\delta}^{x_o+\delta} (\xi - x_o)\; w_5\,(x,\theta,\xi)\; d\xi$$

$$= \sum_{n=1}^{\infty} \bar{F}_{1n}\,(\theta) \left\{ \int_{x_o-\delta}^{x_o+\delta} (\xi - x_o) \sin(\lambda\xi) d\xi \right\} \sin\,(\lambda x) \qquad (15)$$

where

$$\bar{F}_{1n}(\theta)\, \sin\,(\lambda\xi) = F_{1n}(\theta).$$

Since

$$\int_{x_o-\delta}^{x_o+\delta} (\xi - x_o)\, \sin\,(\lambda\xi)\, d\xi = \frac{2}{\lambda^2} \cos\,(\lambda x_o) \left\{ \sin(\lambda\delta) - \lambda\delta\, \cos(\lambda\delta) \right\},$$

it follows that the desired solution for a distribution of longitudinal moment a[illegible] over a series of N pads may be expressed by Equations 11 through 14, provided $F_{mn}(\theta)$, $(m = 1,2,3)$ are modified by being multiplied by the quantity

$$\frac{2}{\lambda^2} \cot (\lambda x_o) \left\{ \sin (\lambda\delta) - \lambda\delta \cos (\lambda\delta) \right\} ,$$

and provided that in $F_{mn}(\theta)$, $p^*(S)$ is given by

$$p^*(S) = F \left\{ p(\theta) \right\} = \frac{1}{\pi S} \left\{ p_o \sin (S\theta_1) \sum_{m=1}^{N} e^{-2(m-1)(\theta_1+\theta_2)iS} \right\}$$

$$\begin{aligned} &= \frac{1}{\pi S} p_o \sin (S\theta_1) \left\{ N \right\} \qquad S = k \cdot N \\ & \qquad\qquad\qquad\qquad\qquad\qquad (k\ 1,2,\ldots) \\ &= 0 \qquad\qquad\qquad\qquad\qquad S \neq k \cdot N . \end{aligned} \qquad (16)$$

NUMERICAL EXAMPLE AND DISCUSSION

By way of illustration of the preceding results, computations have been made for a cylindrical shell with the parameters

$$a = 10'', \qquad L = 35'', \qquad h = 0.04'',$$

$$x_o = 0.8, \qquad E = 10^7 \text{ psi} \qquad \nu = 0.3$$

and having three areas over which the axial load is exerted with parameters $\delta = 0.10$ and $\theta_1 = 0.025$. (These correspond to three brackets, two inches in length and one-half inch in width, located eight inches from the top of the cylinder). Figures 4a, b, c, and d present stress and moment resultants for the case of a self-equilibrating axial load, while Figures 5a, b, c, and d present similar results for the case of a longitudinal moment M acting over the same areas. If such a configuration were used, for example, to transmit into the cylindrical shell a total load (force) of 1500 pounds, and if the distance from the middle surface of the shell to the application of the load were, say, one-quarter inch, then $p_1 = 500$ psi and $M = 125$ in-lb. The following distribution of stresses on the inner and outer surfaces of the cylinder would result along $\theta=0$:

Table I. SURFACE STRESSES (PSI) VERSUS AXIAL DISTANCE (IN.)

ax	$\sigma_{x_{in}}$	$\sigma_{x_{out}}$	$\sigma_{\theta_{in}}$	$\sigma_{\theta_{out}}$
9	-21,662	+20,980	-18,523	+35,909
9-1/2	+522	-734	-8,491	+12,853
10	-300	-1,500	-5,796	+4,946
10-1/2	-1,482	-1,080	-4,025	+2,875
12	-1,637	-1,233	-1,174	+1,314

In Reference 2, Bijlaard presents extensive numerical results for localized loading of cylindrical shells by radial loads and by longitudinal moments. This provides an opportunity to compare the results of Flügge's equations for distributed loads with those employed by Bijlaard for a wide range of shell parameters. Furthermore, the comparison can be made on the basis of actual stresses in the shell rather than on the basis of displacements. Tables II and III typify the results of such comparisons. (In these tables, we employ Bijlaard's notation in which $\alpha = \ell/a$, $\gamma = a/h$, $\beta_1 = \theta_1$, $\beta_2 = \delta$, and for these computations $\beta_1 = \beta_2$). For radial loads, we observe that maximum differences occur when both β and γ are small, and that as γ increases, N_x and N_θ tend to agree; but if $\beta << 1$, results for M_x and M_θ by the two methods may still disagree when γ is quite large.

Table II. NONDIMENSIONAL STRESS RESULTANTS (RADIAL LOAD P)

		aN_x/P		aN_θ/P		M_x/P		M_θ/P	
β	γ	F.	B.	F.	B.	F.	B.	F.	B.
1/128	5	-1.000	-0.9	-0.666	-1.1	0.346	0.390	0.356	0.430
1/128	50	-9.100	-9.0	-9.900	-10.0	0.225	0.275	0.254	0.330
1/128	300	-55.200	-55.0	-57.200	-57.0	0.159	0.175	0.195	0.250
1/8	5	-0.850	-0.8	-0.712	-0.9	0.114	0.112	0.153	0.155
1/8	50	-6.420	-6.5	-3.460	-3.5	0.031	0.031	0.171	0.070
1/8	300	-25.000	-25.0	-5.200	-5.3	0.011	0.011	0.032	0.031

F. Flügge's equations.
B. Bijlaard's equations.

Table III. NONDIMENSIONAL STRESS RESULTANTS (LONGITUDINAL MOMENT M)

β	γ	$a^2\beta N_x/M$		$a^2\beta N_\theta/M$		$a\beta M_x/M$		$a\beta M_\theta/M$	
		F.	B.	F.	B.	F.	B.	F.	B.
1/32	5	-0.048	-0.088	+0.053	-0.033	+0.1070	+0.108	+0.0660	+0.066
1/32	50	-0.464	-0.420	-1.720	-1.800	+0.1010	+0.101	+0.0630	+0.063
1/32	300	-7.550	-7.600	-27.500	-27.500	+0.0780	+0.081	+0.0500	+0.053
1/8	5	-0.096	-0.060	-0.169	-0.250	+0.1000	+0.100	+0.0600	+0.060
1/8	50	-1.860	-1.800	-5.630	-5.600	+0.0540	+0.054	+0.0367	+0.036
1/8	300	-10.300	-10.300	-18.500	-18.500	+0.0098	+0.010	+0.0086	+0.009

F. Flügge's equations.
B. Bijlaard's equations.

ACKNOWLEDGMENT

It is a pleasure to assert our indebtedness to Mr. O. L. Bowie of the staff of AMRA for several illuminating discussions on this problem.

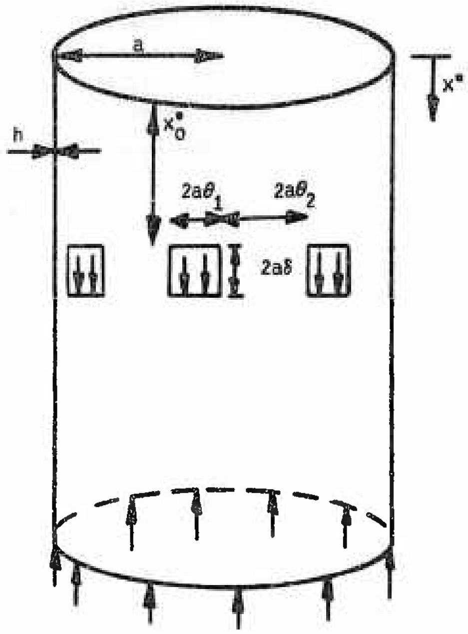


Figure 1. SHELL GEOMETRY

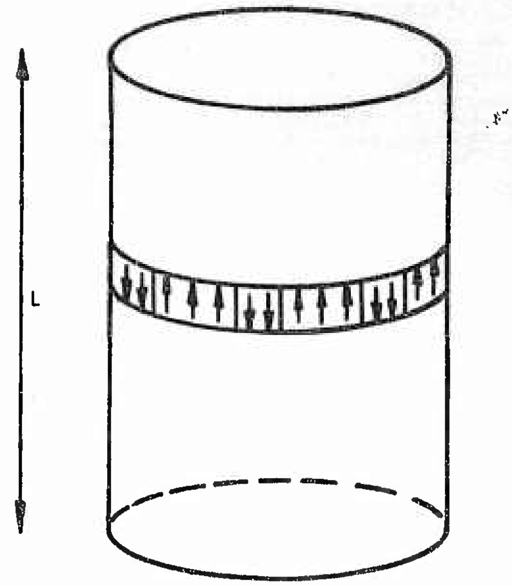


Figure 2. SELF-EQUILIBRATING LOAD SYSTEM $p_x(x,\theta)$

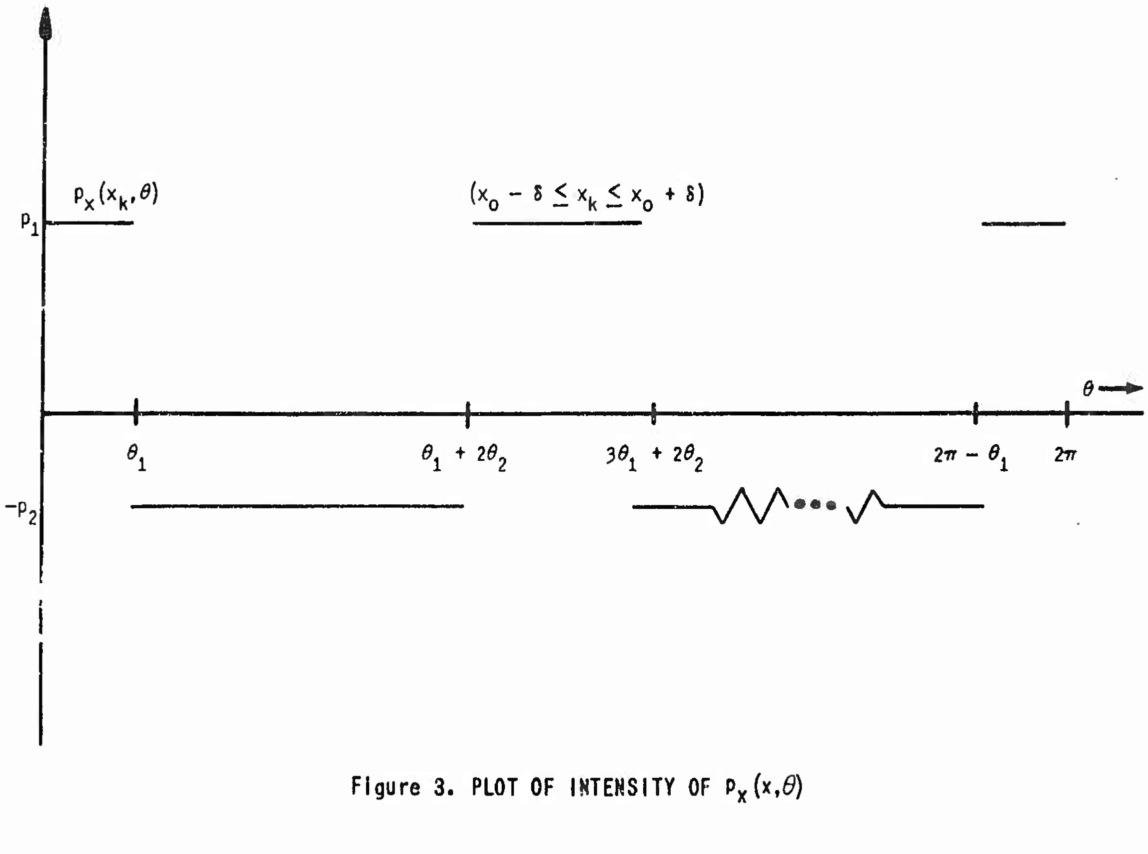
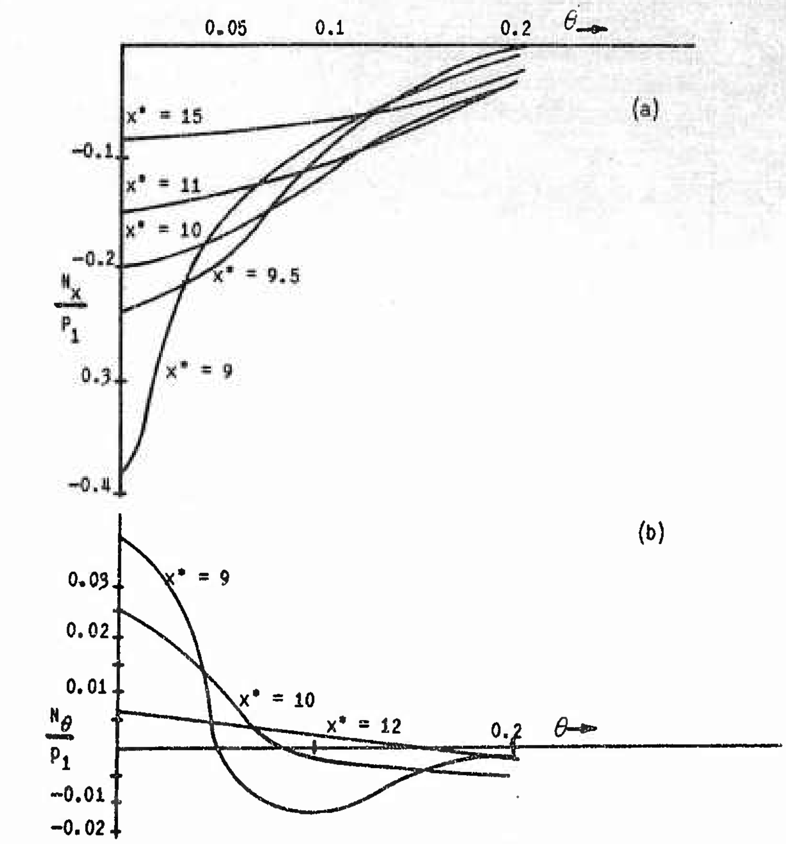
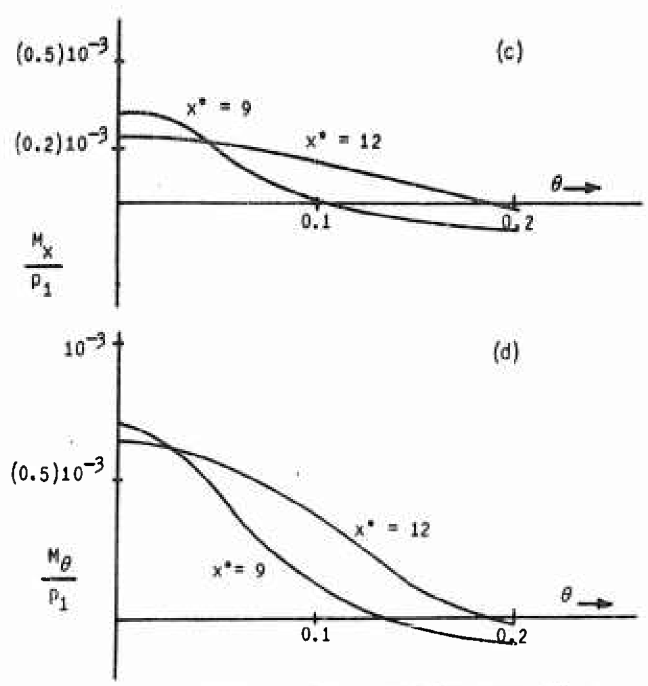


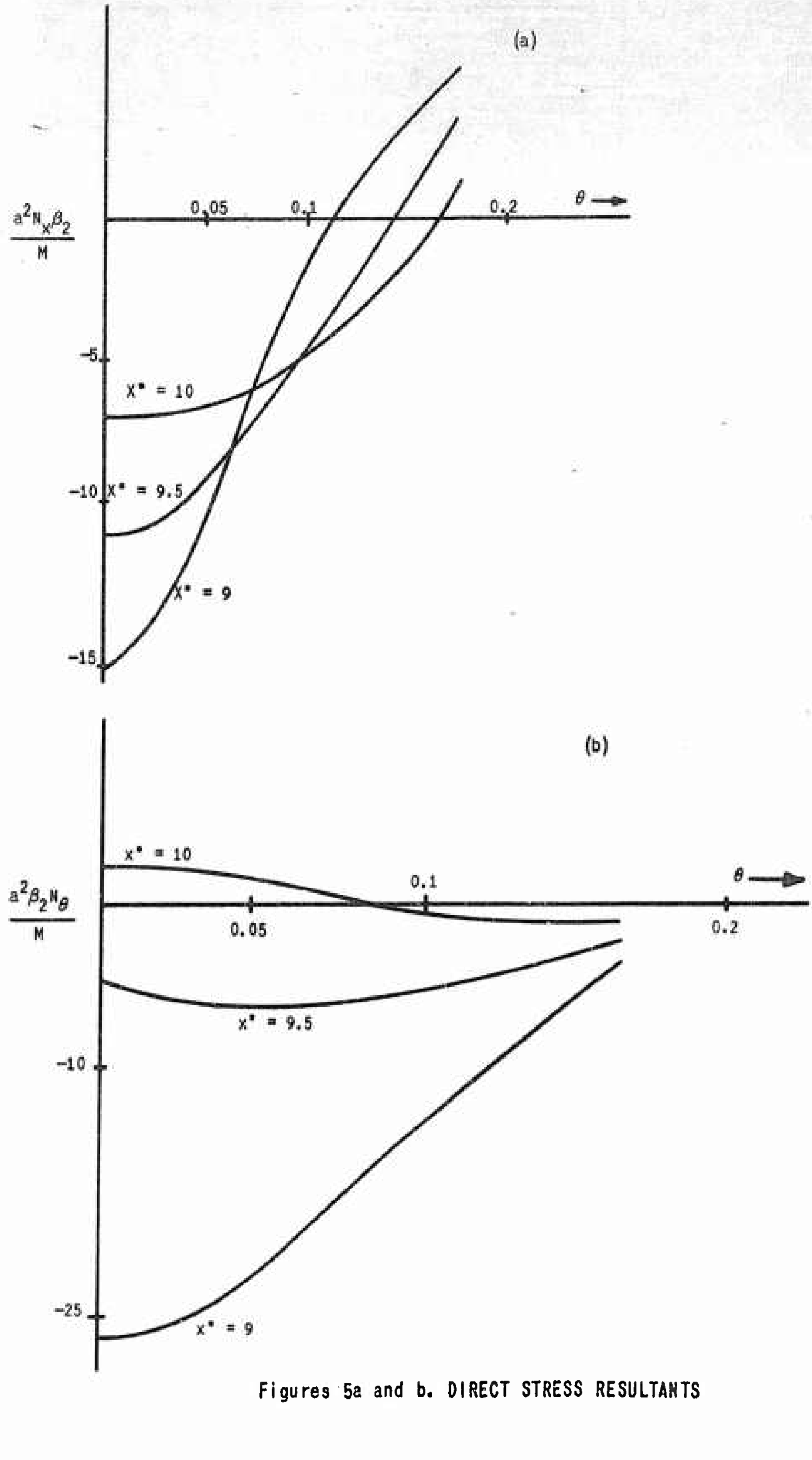
Figure 3. PLOT OF INTENSITY OF $p_x(x,\theta)$



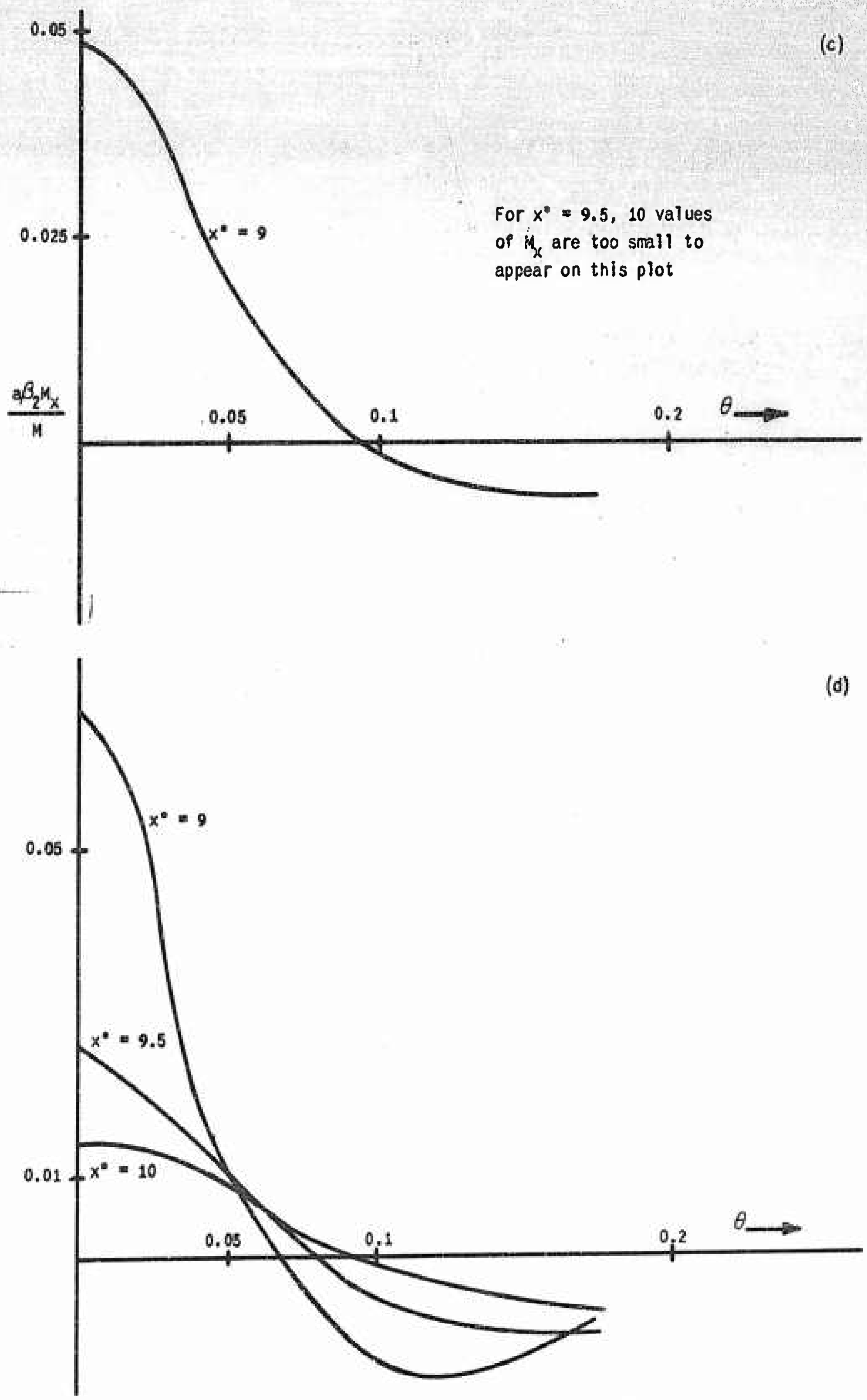
Figures 4 a and b. DIRECT STRESS RESULTANTS



Figures 4c and d. MOMENT RESULTANTS



Figures 5a and b. DIRECT STRESS RESULTANTS



Figures 5c and d. MOMENT RESULTANTS

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The linear equations due to Flügge for thin elastic circular cylindrical shells are solved by use of the finite Fourier transform for the case of distributed tangential (axial) loading with simply supported edge conditions. This solution supplements prior results obtained by Bijlaard for radial (normal) distributed loadings. Combining Bijlaard's results with the solution obtained in this report permits discussion of localized loadings having components both normal and tangential to the shell's middle surface. Such an application is discussed and numerical results are provided.

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